ALTERNATING CURRENT (A.C. CIRCUITS)

Alternating current

An electrical current, magnitude of which changes with time and polarity reverses periodically is called alternating current (A.C)

The sinusoidal alternating current (a.c) is expressed as

$I = I_m \sin \omega t$

where I_m is the maximum value or peak value or amplitude of a.c.

$\omega = \frac{2\pi}{T} = 2\pi f$, where f is called frequency. In India f=50Hz.

Sinusoidal e.m.f. of an a.c. source is given by $\varepsilon = \varepsilon_m \sin \omega t$

where ε_m is the maximum value or peak value or amplitude of e.m.f.

The pictorial symbol used to represent the a.c. source is shown in figure.

Mean or average value of AC

The average value of ac voltages and current over a complete cycle of AC is zero.

Average value of alternating emf and current over a half cycle are $\frac{2\varepsilon_m}{\pi}$ and $\frac{2I_m}{\pi}$ respectively.

ROOT MEAN SQUARE (RMS) OR VIRTUAL OR EFFECTIVE VALUE OF A.C.

The average value of alternating current or emf over a cycle is zero. Therefore we take root mean square value

Root mean square value of alternating current is the direct current which produces the same heating effect in a given resistor in a given time as is produced by the alternating current. It is denoted by I_{rms}, I_v or by I_{eff}.

> If not otherwise mentioned, the values of alternating voltages or currents quoted any where are virtual (r.m.s) values only.
> For example 220*V* a.c. means $V_{rms} = 220\text{volt}$. An ac of 1A means $I_{rms} = 1\text{ampere}$

Relation between the effective value and peak value of a.c.

$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$

Clearly r. m. s value of an alternating current is 70.7% of peak value.

ROOT MEAN SQUARE VALUE OF AN ALTERNATING VOLTAGE

It is defined as the steady voltage that produces the same heating effect in a given resistance in a given time as is produced by the given alternating emf. It is denoted by V_{rms} or V_{eff} or V_v. it is give by, $V_{rms} = \frac{V_m}{\sqrt{2}}$.

Qn. The peak value of an a.c. supply is 300V. What is the rms voltage?

(b) The rms value of current in an a.c circuit is 10A. What is the peak current?

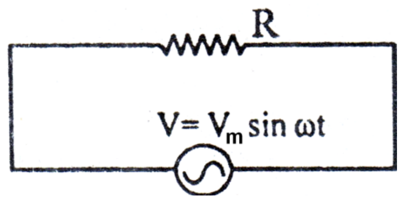
Ans. (a) Here $V_m = 300V$

$\therefore V_{rms} = 0.707 V_m = 0.707 x 300 = 212.1V$

(b) Here $I_{rms} = 10A$

$I_m = \sqrt{2} I_{rms} = 1.414 x 10 = 14.14A$

A.C CIRCUIT CONTAINING ONLY A RESISTOR



Consider a circuit containing a resistance 'R' connected to an alternating voltage.

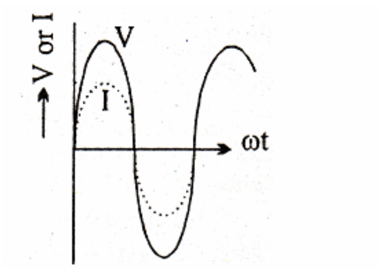
Let the applied voltage be

$V = V_m \sin \omega t$ ....(1)

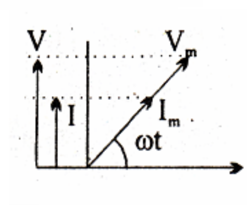
If I be the current in the circuit at instant t, then the potential drop across R will be IR.

According to Kirchhoff's loop rule,

$V_m \sin \omega t = IR$



(a)Graph of V and I versus ωt



(b)Phasor diagram

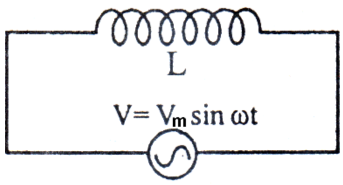
or $I = \frac{V_m}{R} \sin \omega t$

or $I = I_m \sin \omega t$ ....(2)

where $I_m = \frac{V_m}{R}$ =the maximum or peak value of a.c.

From eqn(1) and (2), we can understand that the current and voltage are in same phae

A.C. CIRCUIT CONTAINING ONLY AN INDUCTOR



Consider a circuit containing an inductor of inductance 'L' connected to an alternating voltage. Let the applied voltage be

$V = V_m \sin \omega t$ ....(1)

A back emf $-L\frac{dI}{dt}$ is developed across the inductor. According to Kirchhoff's loop rule

$V_m \sin \omega t - L\frac{dI}{dt} = 0$

or $L\frac{dI}{dt} = V_m \sin \omega t$

or $dI = \frac{V_m}{L} \sin \omega t dt$

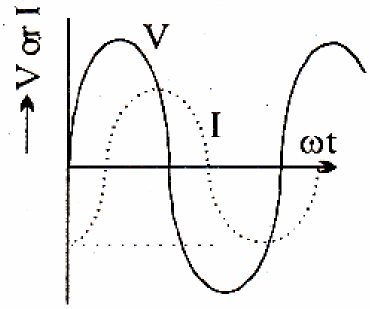
Integrating, $\int dI = \int \frac{V_m}{L} . \sin \omega t . dt$

or $I = -\frac{V_m}{\omega L} \cos \omega t = \frac{V_m}{\omega L} \sin(\omega t - \pi/2) = I_m \sin(\omega t - \pi/2)$ ....(2) $[\therefore -\cos \omega t = \sin(\omega t - \pi/2)]$

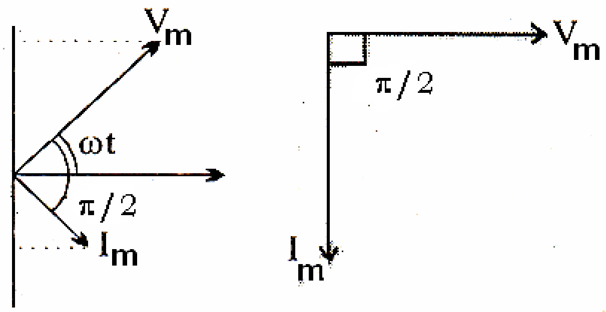
Where $I_m = \frac{V_m}{\omega L}$ =the peak value of a.c.

The term ωL is called inductive reactance (X_L).

On comparing equations (1) and (2), we can understand that, the current lags behind the voltage by an angle $\pi/2$ radian.



(a) Graph of V and I versus ωt



(b)Phasor diagram

Inductive reactance (X_L)

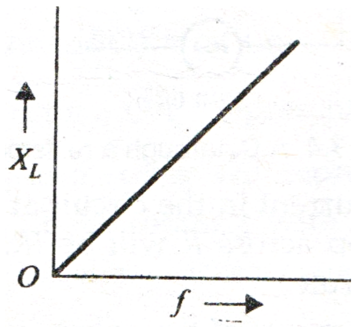
Inductive reactance $X_L = \omega L = 2\pi f L$

where f is the frequency of a.c. supply. The SI unit of inductive reactance is ohm(Ω)

For a.c., $X_L \alpha f$

For d.c., f=0, so $X_L = 0$

Thus an inductor allows flow of d.c through it easily but opposes the flow of a.c. through it.



Graph of X_L versus *f*.

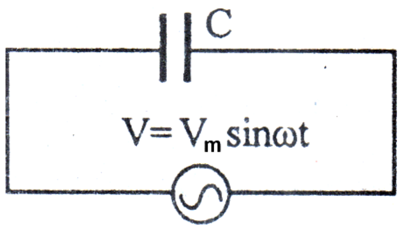
Qn. A 44mH inductor is connected to 220V, 50Hz a.c. supply. Determine the rms value of current in the circuit.

A.C. CIRCUIT CONTAINING ONLY A CAPACITOR

Consider a circuit containing a capacitor of capacitance 'C' connected to alternating voltage.

Let the applied voltage be

$V = V_m \sin\omega t$ ....(1)



At any instant, voltage V across the capacitor is $V = \frac{q}{C}$

According to Kirchhoff's loop rule

$\frac{q}{C} = V_m \sin\omega t$

or $q = CV_m \sin\omega t$

$\therefore$ Current at any instant is

$i = \frac{dq}{dt} = \frac{d(CV_m \sin\omega t)}{dt} = \omega CV_m \cos\omega t$

or $I = I_m \cos\omega t$

or $I = I_m \sin(\omega t + \pi/2)$ ....(2)

where $I_m = \omega CV_m = \frac{V_m}{1/\omega C}$ =the current amplitude.

The term $\frac{1}{\omega C}$ is called capacitive reactance (X_C).

Capacitive reactance

Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

The S.I unit of capacitive reactance is ohm(Ω].

For a.c., $X_C \,\alpha \frac{1}{f}$

For d.c., f=0 $\therefore X_C = \infty$

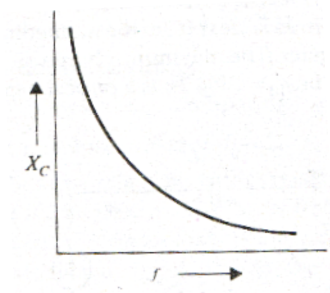
Thus a capacitor blocks d.c.

Variation of capacitive reactance with frequency

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

i.e., $X_C \alpha \frac{1}{f}$

Thus the capacitive reactance varies inversely with the frequency. As f increases, X_C decreases. Figure shows the variation of X_C with f.



Graph of X_C versus *f*.

Qn. A $60\mu F$ capacitor is connected to a 110V, 60Hz a.c. supply. Determine the rms value of current in the circuit.

Qn. While performing an experiment in laboratory, a student connected an electric bulb and a capacitor is series to a source of direct current (dc). Then will be the bulb glow steadily? Explain.

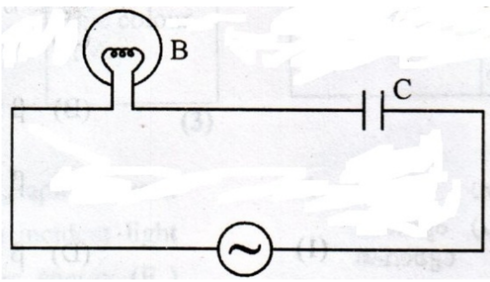
Qn. What happens when

(a) a capacitor is included in a dc circuit.

(b) What type of current easily passes through an inductor?

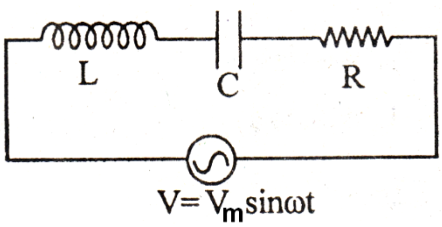
(c) Give an explanation for your answer.

Qn. An electric bulb 'B' and a parallel plate capacitor 'C' are connected in series as shown in figure. The bulb glows with some brightness. How will the glow of the bulb affected on introducing a dielectric slab between the plates of the capacitor? Give reason to support yours answer.



A. C. VOLTAGE APPLIED TO A SERIES LCR CIRCUIT

Let an alternating voltage $V = V_m \sin\omega t$ applied to a circuit containing an inductance L, a capacitance C and resistance R connected in series.



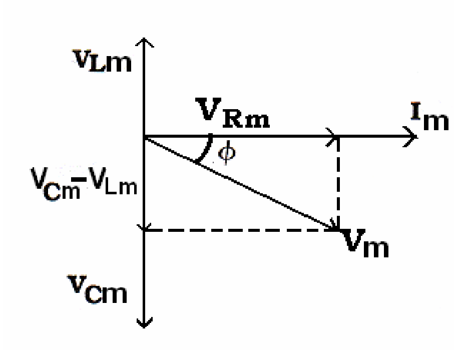
According to Kirchhoff's loop rule

$$V_m \sin\omega t = L\frac{dI}{dt} + IR + \frac{q}{C}$$

The solution to this equation can be obtained by different methods.

Phasor-diagram solution

The phasor diagram for series LCR circuit to which an AC voltage $V = V_m \sin\omega t$ is applied is shown below. The current in each element is same. Voltage across the resistor is in phase with current. But the voltage across the inductor L leads the current by $\frac{\pi}{2}$ radian and voltage across the capacitor C lags behind the current by $\frac{\pi}{2}$. Let peak value of current in the circuit be I_m. Let V_{Rm}, V_{Lm} and V_{Cm} be the peak value of voltage across the resistor, inductor and capacitor respectively. The vector sum of V_{Rm}, V_{Cm} and V_{Lm} gives the peak value V_m of the applied voltage.



By Pythagorean theorem , $V_m^2 = V_{Rm}^2 + (V_{Cm} - V_{Lm})^2$

Substituting the values of V_{Rm}, V_{Cm} and V_{Lm}, we have

$$V_m^2 = (I_m R)^2 + (I_m X_C - I_m X_L)^2$$
$$= I_m^2\left[R^2 + (X_C - X_L)^2\right]$$

or, $$I_m = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

Clearly, $\sqrt{R^2 + (X_C - X_L)^2}$ is the effective resistance of the series LCR circuit to the flow of ac. It is called impedence. It is denoted by Z and its S.I unit is ohm (Ω].

Thus, impedence $Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$

Phase difference

Let ϕ be the phase difference between the voltage and current, from figure we can write

$$\tan\phi = \frac{V_{Cm} - V_{Lm}}{R} = \frac{I_m X_C - I_m X_L}{R} = \frac{X_C - X_L}{R}$$

Special cases

i. If $X_C > X_L$, then current leads the voltage.

ii. If $X_L < X_C$, then the current lags the voltage.

iii. If $X_L = X_C$, then current is in phase with the voltage.

RESONANCE

The current amplitude in an LCR circuit is given by, $I_m = \dfrac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$

As angular frequency ω of alternating emf is increased, $\frac{1}{\omega C}$ goes on decreasing and ωL goes on increasing. For a particular value of $\omega (= \omega_0, \text{say})$ $\frac{1}{\omega C}$ becomes equal to ωL. Then the impedence is minimum ($Z = \sqrt{R^2 + 0} = R$) and the current amplitude becomes maximum ($I_m = \frac{V_m}{R}$]. This phenomenon is known as resonance and the angular frequency ω_0 is called natural or resonant angular frequency.

Determination of resonant frequency

At resonant angular frequency ω_0,

$$X_L = X_C$$

or $\omega_0 L = \frac{1}{\omega_0 C}$

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}}$$

The resonant frequency $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

Characterestics of series resonant circuit

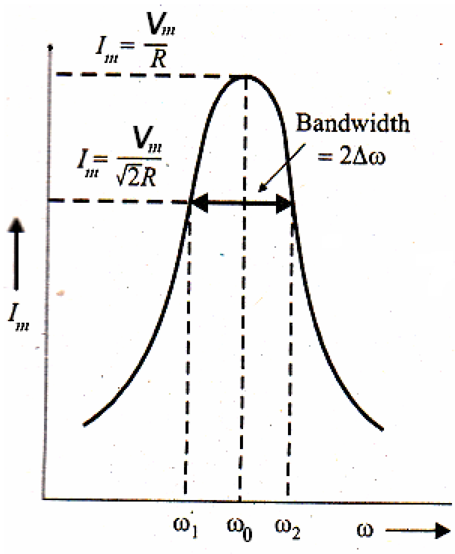
1. The current is in phase with voltage and power factor is unity (cosϕ=1 when $\phi = 0$)
2. The voltage across R is equal to the applied voltage.

Qn. A sinusoidal voltage of peak value 283V and frequency 50Hz is applied to a series LCR circuit in which R=3Ω, L=25.48mH, and C=796μF. Find the impedence of the circuit, (b) the phase difference between the voltage across the source and the currents, (c) the power dissipated in the circuit, and (d) the power factor.

SHARPNESS OF RESONANCE: Q-Factor

The sharpness of resonance is measured by a quantity called the quality or Q-factor of the circuit.

The Q-factor of a series resonant circuit is defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency at which the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value at resonant frequency.



Bandwidth of a series resonant circuit.

Mathematically Q-factor can be expressed as

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\text{Resonant frequency}}{\text{Band width}}$$

where ω_1 and ω_2 are the frequencies at which the current falls to $\frac{1}{\sqrt{2}}$ times its resonant value, as shown in figure.

Q-factor is a dimensionless quantity.

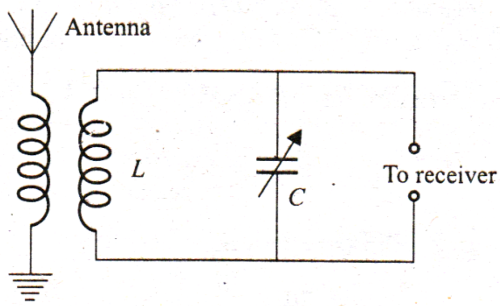
Thus, Q-factor of a series LCR-circuit may also be defined as the ratio of either the inductive reactance or the capacitive reactance at resonance to the resistance of the circuit.

$$Q = \frac{\omega_0 L}{R} = \frac{(1/\omega_0 C)}{R}$$

When R is low, Q is high and greater will be the sharpness of resonance.

Tuning of a radio receiver

The tuning circuit of a radio or TV is an example of LCR resonant circuit. Signals are transmitted by different stations at different frequencies. These frequencies are picked up by the antenna and corresponding to these frequencies, a number of voltages appear across the series LCR-circuit. But maximum current flows through the circuit for that a.c. voltage which has frequency equal to $f_0 = \frac{1}{2\pi\sqrt{LC}}$. If the Q-value of the circuit is large, the signals of the other stations will be very weak i.e., circuit will be more selective. By changing the value of the adjustable capacitor C, signal from the desired station can be tuned in.



Series resonant circuit.

AVERAGE POWER IN AN AC CIRCUIT

Suppose in an a.c circuit, the voltage and current at any instant are given by

$$V = V_m \sin \omega t$$

and $I = I_m \sin(\omega t + \phi)$
where ϕ is the phase angle by which current I leads the voltage V.
The instantaneous power is given by
$P = VI = V_m I_m \sin(\omega t + \phi).\sin\omega t$

$= \frac{V_m I_m}{2} 2\sin(\omega t + \phi)\sin\omega t = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$ $[\because 2\sin A \sin B = \cos(A - B) - \cos(A + B)]$

The average power in the circuit over a complete cycle is

$$P_{av} = \frac{1}{T}\int_0^T P dt = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t + \phi)]dt = \frac{V_m I_m}{2T}\left[\int_0^T \cos\phi dt - \int_0^T \cos(2\omega t + \phi)dt\right]$$

$$= \frac{V_m I_m}{2T}\left[\cos\phi\int_0^T dt - 0\right] \qquad \left(\therefore \int_0^T \cos(2\omega t + \phi)dt = 0\right)$$

$$= \frac{V_m I_m}{2T}[\cos\phi(T - 0)]$$

or $P_{av} = \frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}\cos\phi$

or $P_{av} = V_{rms} I_{rms} \cos\phi$

Special cases

1. Pure resistive circuit

Here voltage and current are in same phase, i.e., $\phi = 0$

$\therefore P_{av} = V_{rms}.I_{rms}\cos 0 = V_{rms} I_{rms}$

2. Pure inductive circuit

Here voltage leads the current in phase by $\frac{\pi}{2}$, i.e., $\phi = \frac{\pi}{2}$

$\therefore P_{av} = V_{rms} I_{rms} \cos\frac{\pi}{2} = 0$

Thus the average power consumed in an inductive circuit over a complete cycle is zero.

3. Pure capacitive circuit

Here voltage lags behind the current in phase by $\frac{\pi}{2}$,i.e., $\phi = -\frac{\pi}{2}$

$\therefore P_{av} = V_{rms} I_{rms} \cos\left(-\frac{\pi}{2}\right) = 0$

Thus the average power consumed in a capacitive circuit over a complete cycle is also zero.

POWER FACTOR

The average power of an a.c. circuit is given by
$P_{av} = V_{rms} I_{rms} \cos\phi$

The product $V_{rms} I_{rms}$ does not give the actual power and is called apparent power. The factor $\cos\phi$ is called power factor of an a.c. circuit.

$\therefore$ True power=Apparent power x Power factor

Thus power factor may be defined as the ratio of true power to the apparent power of an a.c. circuit.

Wattless current (Idle current)

If the average power consumed in an a.c. circuit is zero, then the current in a.c. circuit is said to be wattless. This happens in the case of a pure inductor or capacitor. The current is called wattles because the current in the circuit does not do any work.

AVERAGE POWER ASSOCIATED WITH A RESISTOR

$V = V_m \sin\omega t$ and $I = I_m \sin\omega t$

Instantaneous power $P = VI = V_m I_m \sin^2\omega t = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$

Hence the average power consumed by the resistor over a complete cycle (i.e., from t=0 to t=T) is

$$P_{av} = \frac{1}{T}\int_0^T P dt = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}(1-\cos 2\omega t)dt = \frac{V_m I_m}{2T}\int_0^T (1-\cos 2\omega t)dt = \frac{V_m I_m}{2T}\left[\int_0^T dt - \int_0^T \cos 2\omega t dt\right]$$

$$= \frac{V_m I_m}{2T}[T-0] \qquad \left[\because \int_0^T \cos 2\omega t dt = 0\right]$$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}} = V_{rms}.I_{rms} = \frac{V_{rms}^2}{R} \qquad \left[\because I_{rms} = \frac{V_{rms}}{R}\right]$$

AVERAGE POWER ASSOCIATED WITH A PURE INDUCTOR

$V = V_m \sin\omega t$

and $I = I_m \sin\left(\omega t - \frac{\pi}{2}\right) = -I_m \cos\omega t$

Instantaneous power,

$$P = VI = -V_m I_m \sin\omega t \cos\omega t = -\frac{V_m I_m 2\sin\omega t\cos\omega t}{2} = -\frac{V_m I_m}{2}\sin 2\omega t$$

Average power over a complete cycle,

$$P_{av} = \frac{1}{T}\int_0^T P dt = \frac{1}{T}\int_0^T -\frac{V_m I_m}{2}\sin 2\omega t dt = -\frac{V_m I_m}{2T}\int_0^T \sin 2\omega t dt$$

$$=0 \left[\because \int_0^T \sin 2\omega t dt = 0\right]$$

Thus average power dissipated per cycle in an inductor is zero.

> During the first quarter of each current cycle, as the current increases, the magnetic flux through the inductor builds up and energy is stored in the inductor from the external source. In the next quarter of cycle, as the current decreases, the flux decreases and the stored energy is returned to the source. Thus, in half cycle, no net power is consumed by the inductor.

AVERAGE POWER ASSOCIATED WITH A CAPACITOR

$V = V_m \sin\omega t$

and $I = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = I_m \cos\omega t$

Instantaneous power $P = VI = V_m I_m \sin\omega t \cos\omega t$

$$= \frac{V_m I_m}{2}\sin 2\omega t$$

Average power over a complete cycle

$$P_{av} = \frac{1}{T}\int_0^T P dt = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\sin 2\omega t dt$$

$$= \frac{V_m I_m}{2T}\int_0^T \sin 2\omega t dt = 0 \left[\because \int_0^T \sin 2\omega t dt = 0\right]$$

> When the capacitor is connected across an a.c. source, it absorbs energy from the source for a quarter cycle as it is charged. It returns energy to the source in the next quarter cycle as it is discharged. Thus in a half cycle, no net power is consumed by the capacitor.

Choke coil

Choke coil is simply an inductor. Choke coil offers a reactance $X_L = 2\pi fL$ to the flow of alternating current. Average power consumed by a choke coil is zero. Thus, a choke coil reduces current in an a.c. circuit without consuming any power.

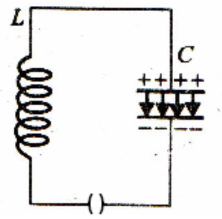
When an ohmic resistance is used, current reduces but energy loss occur due to heating. So choke is preferred.

For d.c, f=0, so $X_L = 0$ i.e., choke coil cannot limit direct current.

Qn. A choke coil and a bulb are connected in series to an a.c. source. The bulb shines brightly. How does its brightness change when an iron core is inserted in the choke coil?

Ans. When the iron core is inserted in the choke coil, the self-inductance L increases. Consequently, the inductive reactance, $X_L = \omega L$ increases. This decreases the current in the circuit and bulb glows dimmer.

LC-OSCILLATIONS

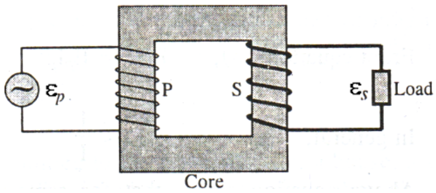


When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called LC-oscillations.

The frequency of oscillation of charge or current $f = \frac{1}{2\pi\sqrt{LC}}$

However, the LC oscillations are usually damped due to the resistive losses in the inductor and dielectric losses in the capacitor.

TRANSFORMER: Transformer is a device used to convert low alternating voltage at higher current into high alternating voltage at low current and vice-versa. in other words, a transformer is an electrical device used to increase or decrease alternating voltage.



Construction

It consists of two separate coils of insulated wire wound on same iron core. One of the coils connected to a.c. input is called primary(P) and the other winding giving output is called secondary (S).

Theory

When an alternating source of emf ε_p is connected to the primary coil, an alternating current flows through it. Due to the flow of alternating current magnetic flux linked with the primary and secondary changes. This produces an emf in the secondary. Let N_P and N_S be the number of turns of primary and secondary coils repectively. The iron core is capable of coupling the whole of the magnetic flux ϕ produced by the turns of the primary coil with the secondary coil.

According to Faraday's law of electromagnetic induction, the induced emf in the primary coil,

$\varepsilon_p = -N_P \frac{d\phi}{dt}$ ....(1)

The induced emf in the secondary coil,

$\varepsilon_s = -N_S \frac{d\phi}{dt}$ ....(2)

Dividing (2) by (1), we get

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$$

If output voltage is less than the input voltage, the transformer is called step down transformer. In a step down transformer $N_s < N_p$ and $\varepsilon_s < \varepsilon_p$.

If the output voltage is greater than the input voltage, the transformer is called step up transformer. In a step up transformer $N_s > N_P$ and $\varepsilon_s > \varepsilon_p$.

For an ideal transformer(in which there are no energy losses), Output power=Input power

$\therefore \varepsilon_s I_s = \varepsilon_p I_p$

For a transformer, efficiency, $\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\varepsilon_s I_s}{\varepsilon_p I_p}$

For an ideal transformer, efficiency, η is 100%. But in a real transformer, the efficiency varies from 90-99%. This indicates that there are some energy losses in the transformer.

Energy losses in transformers

1. Copper loss. It is energy loss due to heating of the copper windings due their resistance.

2. Eddy current loss. It is the energy loss due to heating of the core by the eddy current. This loss can be reduced by using laminated iron core.

3. Hysteresis loss. It is the energy loss due to the heating of the core due to the application of cyclic magnetizing field. It can be minimized by using core material having narrow hysteresis loop.

4. Flux leakage. The magnetic flux produced by the primary may not fully pass through the secondary. This loss can be minimized by winding the primary and secondary coils over one another.